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Effect of Lining Anisotropy on Sound Attenuation in Lined Ducts

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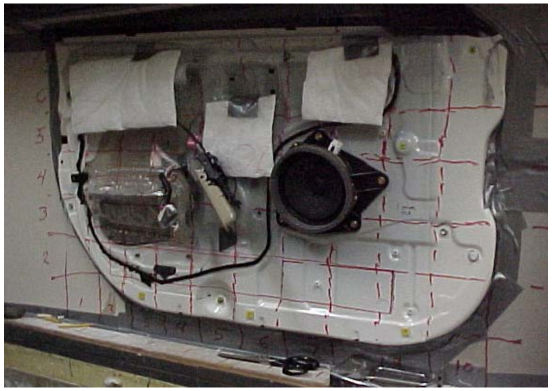
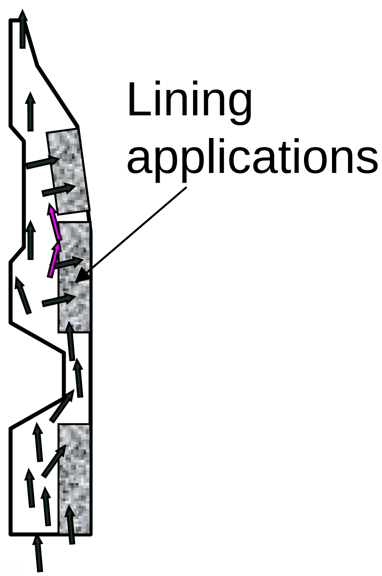
Introduction

Car door system





Grazing Incidence Applications

- Porous materials are often used to line "channels"
- previous work show that the properties that contral grazing direction attenuation are somewhat different from those that contral normal absorption.

Here introduce anisotropic theory to account for different flow resistivity of the lining material in normal and grazing directions.

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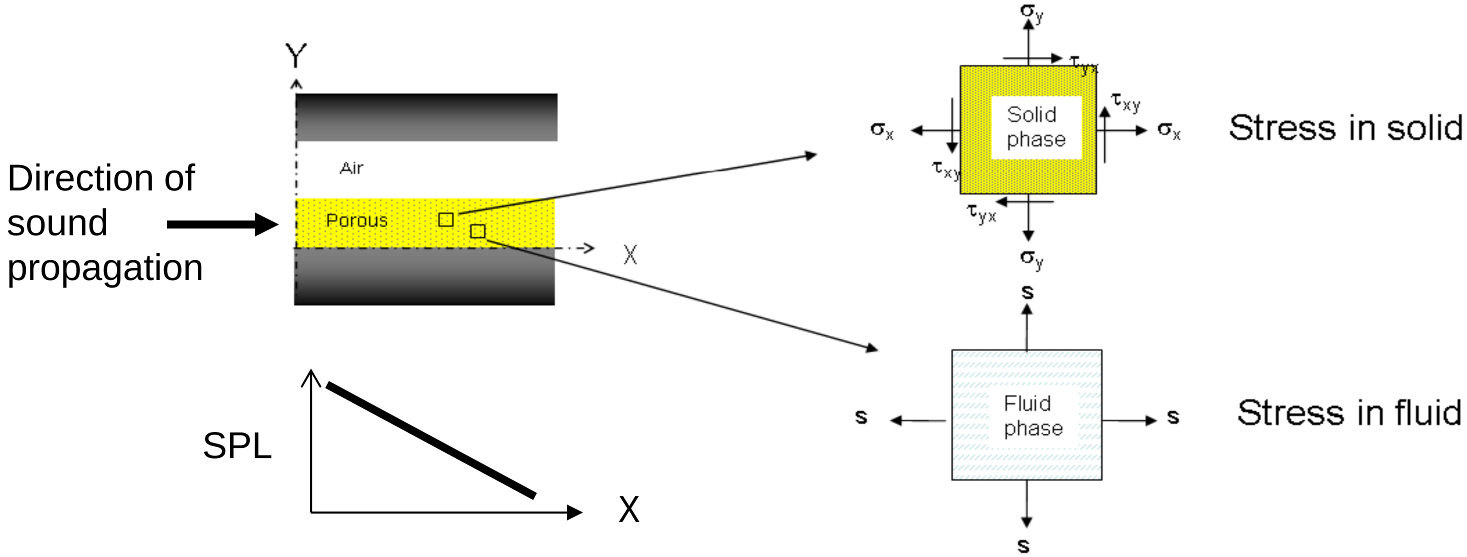
Duct lining geometry and coordinate system for poro-elastic lining



Transversely isotropic poro-elastic displacement and stress field

$$u_x = e^{-jk_x x}\left(\sum_{i=1}^{4}\alpha_i C_i e^{-jk_{iz}z} + \sum_{i=5}^{6} C_i e^{-jk_{iz}z}\right)$$

$$u_z = e^{-jk_x x}\left(\sum_{i=1}^{4} C_i e^{-jk_{iz}z} + \sum_{i=5}^{6}\alpha_i C_i e^{-jk_{iz}z}\right)$$

$$U_x = e^{-jk_x x}\left(\sum_{i=1}^{6}\beta_i C_i e^{-jk_{iz}z}\right)$$

$$U_z = e^{-jk_x x}\left(\sum_{i=1}^{6}\gamma_i C_i e^{-jk_{iz}z}\right)$$

$$\sigma_z = -je^{-jk_x x}\left[\sum_{i=1}^{4}\{k_x(F\alpha_i + Q\beta_i) + k_{iz}(C + Q\gamma_i)\}C_i e^{-jk_{iz}z} + \sum_{i=5}^{6}\{k_x(F + Q\beta_i) + k_{iz}(C\alpha_i + Q\gamma_i)\}C_i e^{-jk_{iz}z}\right]$$

$$\tau_{xz} = -je^{-jk_x x}\left[\sum_{i=1}^{4}\{k_x(F\alpha_i + Q\beta_i) + k_{iz}(C + Q\gamma_i)\}C_i e^{-jk_{iz}z} + \sum_{i=5}^{6}\{k_x(F + Q\beta_i) + k_{iz}(C\alpha_i + Q\gamma_i)\}C_i e^{-jk_{iz}z}\right]$$

$$s = -je^{-jk_x x}\left[\sum_{i=1}^{4}\{k_x(F\alpha_i + Q\beta_i) + k_{iz}(C + Q\gamma_i)\}C_i e^{-jk_{iz}z} + \sum_{i=5}^{6}\{k_x(F + Q\beta_i) + k_{iz}(C\alpha_i + Q\gamma_i)\}C_i e^{-jk_{iz}z}\right].$$

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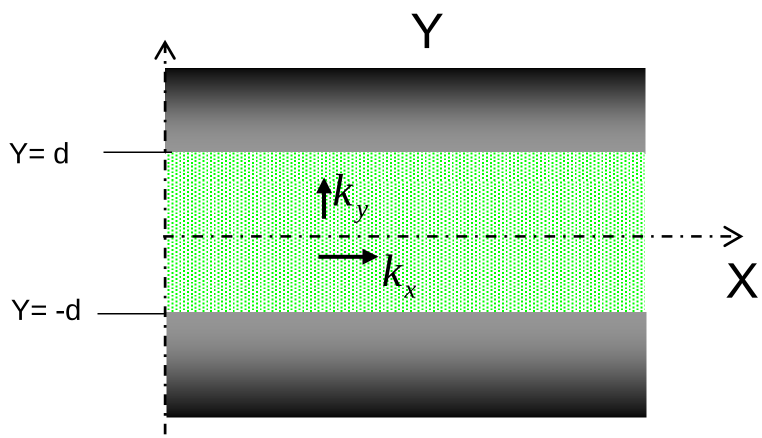
Boundary condition: fully-lined model



- 2-D porous formulation of the free-wave solution in a lined duct.
- Rigid wall boundary condition
- Simplified by symmetric condition
- Solve for k_x to find attenuation rate in lining.

$at\ y=-d$	$at\ y=d$
$(1)\ u_{x(symm)}=0$	$(4)\ u_{x(symm)}=0$
$(2)\ u_{y(symm)}=0$	$(5)\ u_{y(symm)}=0$
$(3)\ U_{y(symm)}=0$	$(6)\ U_{y(symm)}=0$

$$\mathbf{A}_f\mathbf{X}_f=0$$

$$\mathbf{X}_f=\begin{bmatrix}C_1^* & C_3^* & C_5^*\end{bmatrix}^T$$

- The characteristic dispersion equation

$$\det(\mathbf{A}_f)=0$$

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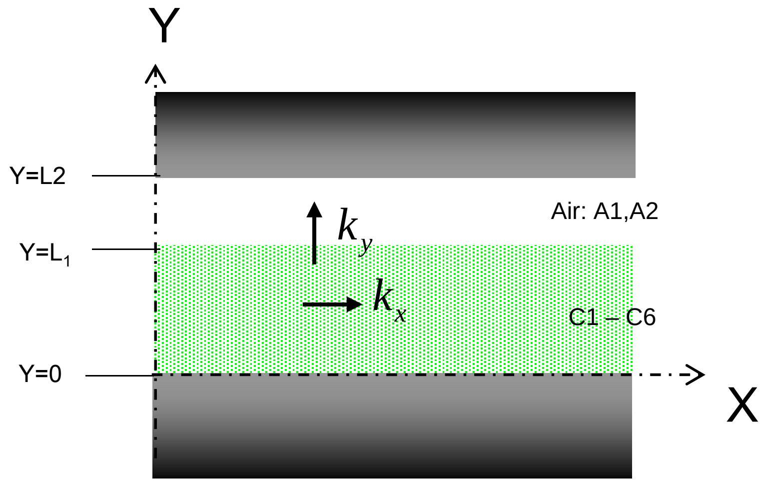
Boundary condition: partially-filled model (FA)



- 2-D porous formulation of the free-wave solution in a lined duct
- Rigid wall boundary condition
- Solve for k_x to find attenuation rate in lining.

$at\ y = 0$

(1) $u_{x,1} = 0$

(2) $u_{y,1} = 0$

(3) $U_{y,1} = 0$

$at\ y = L_1$

(4) $-\Omega_p P = s$

(5) $-(1-\Omega_p)P = \sigma_y$

(6) $v_y = j\omega(1-\Omega_p)u_y + j\omega\Omega_p U_y$

(7) $\tau_{yx} = 0$

- The characteristic dispersion equation

$$\det(\mathbf{A}_p) = 0$$

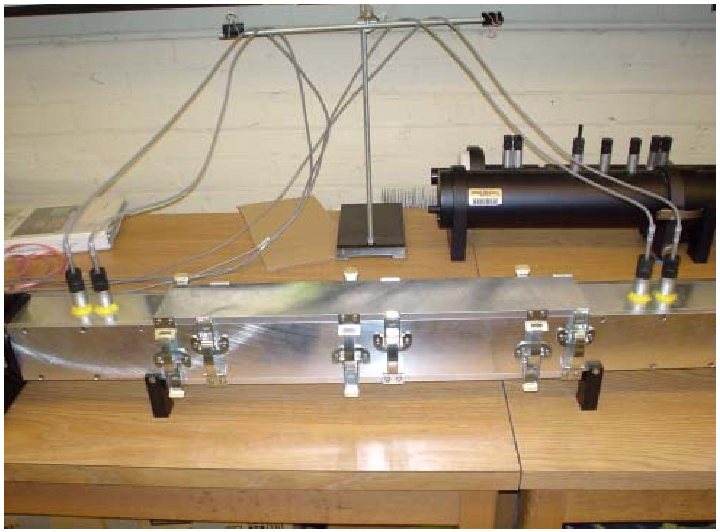
$at\ y = L_2$

(8) $v_y = 0$

$$\mathbf{A}_p \mathbf{X}_p = 0$$

$$\mathbf{X}_p = \begin{bmatrix} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & A_1 & A_2 \end{bmatrix}^T$$

Square duct system and glass fiber layer



0.5m

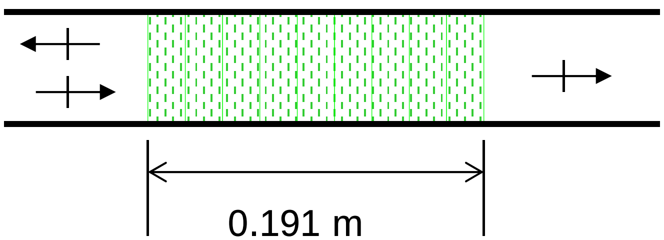
Sample length

• 4-microphone transfer matrix method used to measure attenuation at grazing incidence

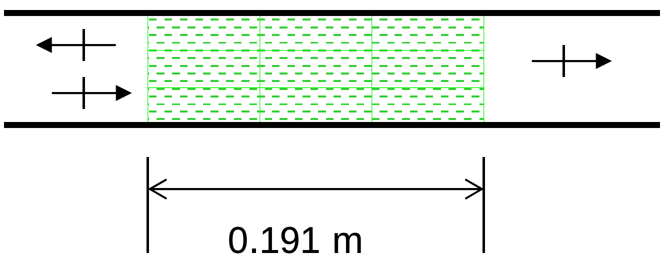
Effect of Material Anisotropy

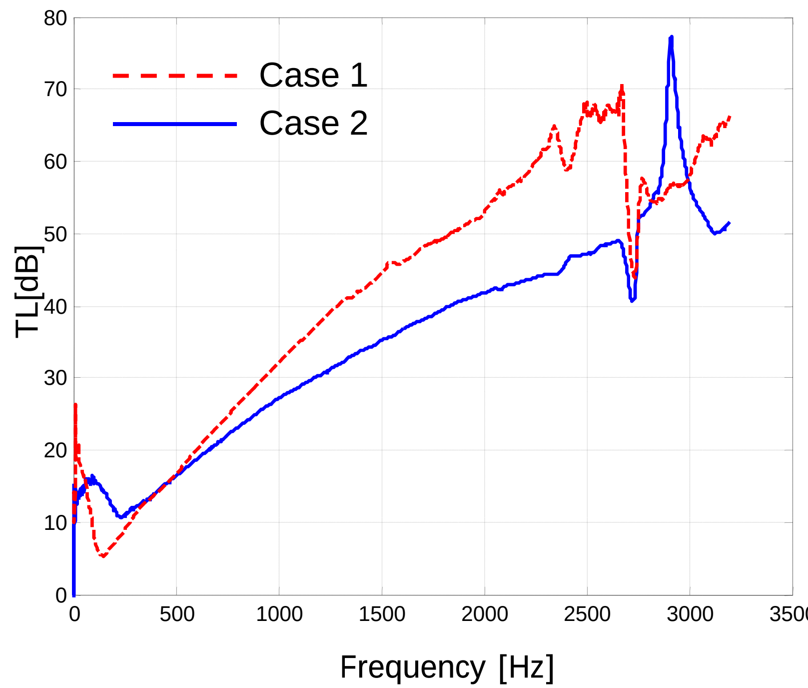
Measurement of TL in two configurations for glass fiber (green)

Case 1:
9 pieces layered in vertical direction



Case 2:
9 pieces layered in horizontal direction





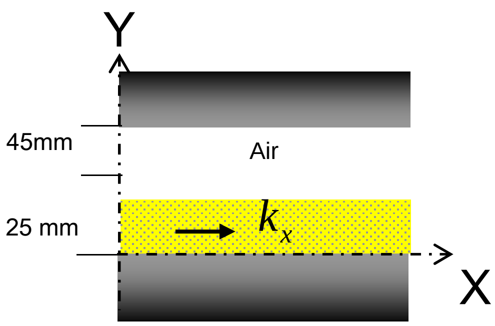
- Acoustic properties depend on material orientation – flow resistivity larger in normal direction

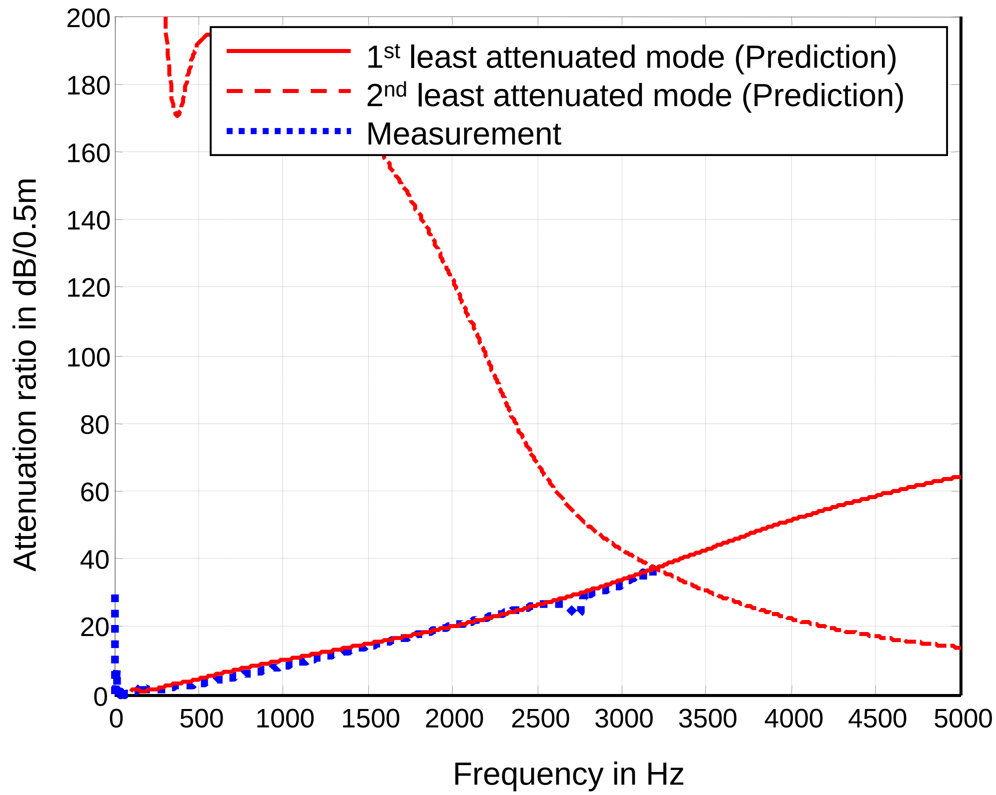
Anisotropic Duct Lining

- **Prediction and measurement for Yellow glass fiber by using anisotropic poro-elastic model**

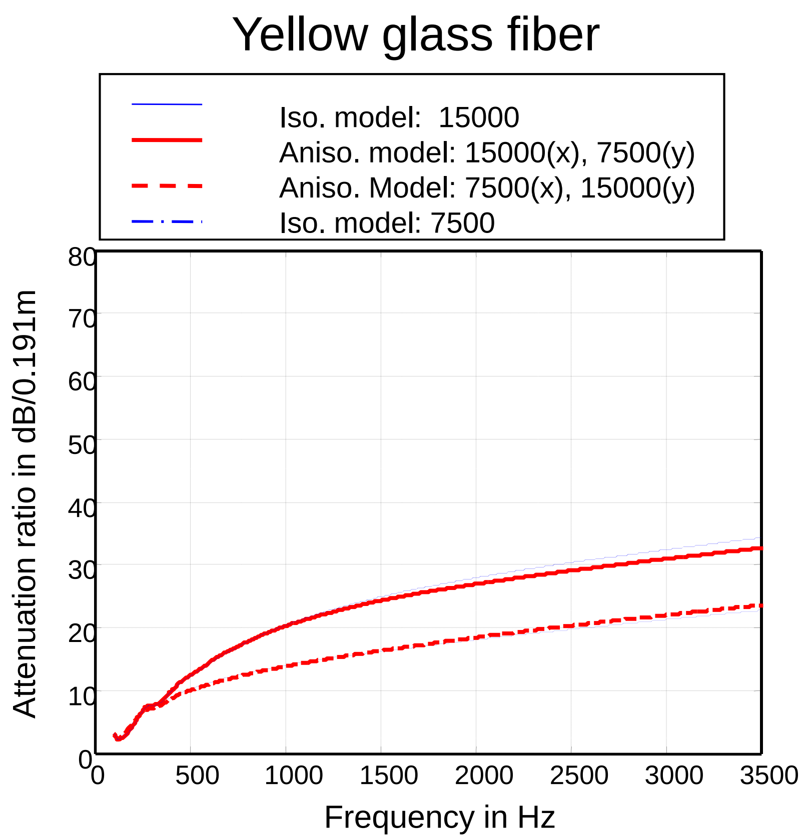
- Different flow resistivities [MKS Rayls/m] in x- and y-directions

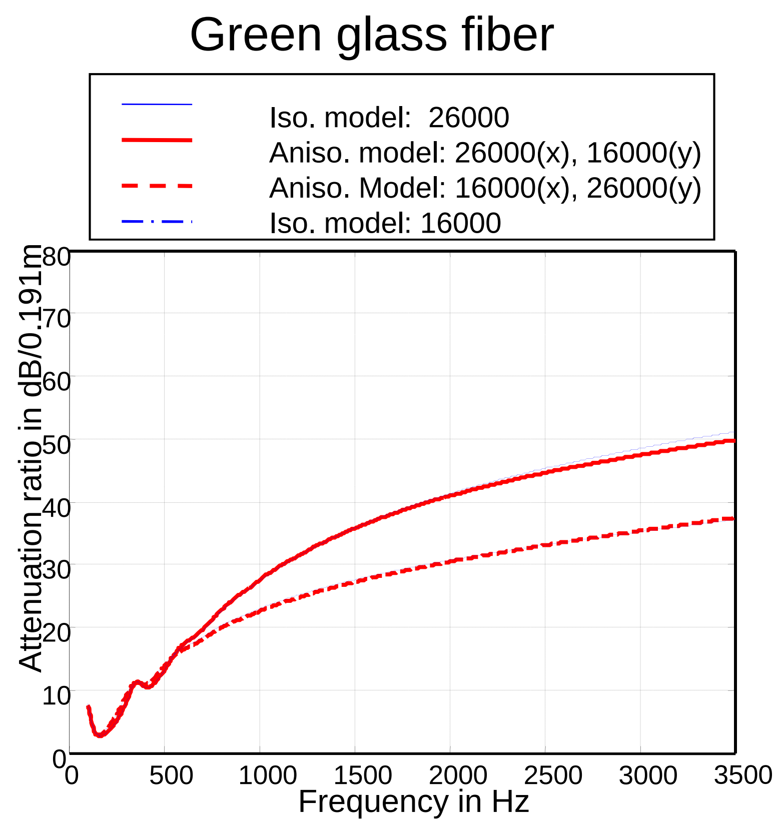
$$\sigma_x = 7500 \quad \sigma_y = 15000$$

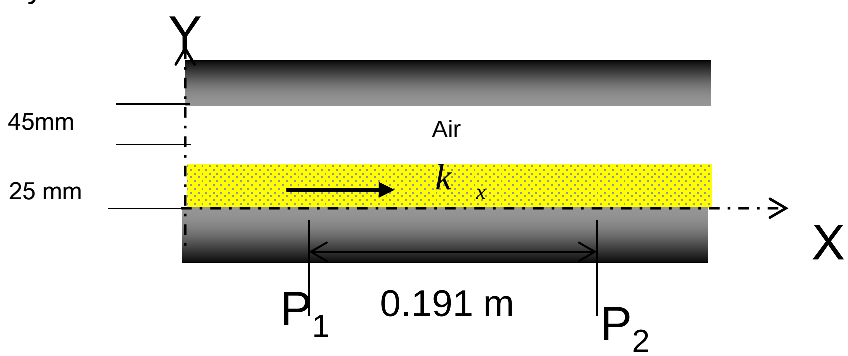




Sensitivity to transverse flow resistivity







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Conclusions

- A free wave propagation model based on a transversely isotropic poro-elastic theory was implemented for both fully-lined and partially-lined duct systems
- Performed experimental measurements of lined duct attenuation rates in *x*- and *y*-directions.
- Showed good agreement between predictions and measurements for partially-lined duct case.
- The study of flow resistivity shows that the *x*-direction is more effective than that of y-direction for controlling the sound attenuation performance in the duct system.
- Comparison of isotropic and anisotropic models shows that anisotropic model is more effective for predicting anisotropic real behavior of the lining materials.